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Influence of Active Damping on a High-Static-Low-Dynamic Stiffness (HSLDS) Isolation System

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ABSTRACT

This paper presents the influence of active damping on the dynamic response of a high- static-low-dynamic stiffness (HSLDS) isolation system. First, a model of a single-degree- of-freedom (SDOF) HSLDS isolation system is presented, and the approximate solution to the equation of motion is determined by the Harmonic Balance method (HBM), to the first order expansion. Then, the effect of active damping on the motion transmissibility performance is studied, accompanied by a comparison with passive damping to demonstrate the advantages of the actively damped HSLDS isolation system. In particular, the boundary of the stability region of the system is determined based on Floquet's theory to demonstrate the effect of active damping on the system transmissibility performance. The results have shown that the increment of active damping produces more dynamically stable system.

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INTRODUCTION

Undesirable vibration always become a challenging issue for many mechanical structures. It is not only affecting the equipment functionality, but also may has adverse effects on human comfort and safety. One of the common methods in reducing undesired vibration is by inserting an isolator between the vibration source and the receiver. Its performance can be improved by having soft stiffness, such that the isolation bandwidth increases. However, if the isolator is linear, this design will result a proportionately higher static deflection issue [1].

Recently, passive nonlinear isolators that has High-Static-Low-Dynamic-Stiffness (HSLDS) characteristic have been considered by many researchers [2-3]. This is due to its capability in obtaining wide isolation bandwidth frequency by lowering the natural frequency of the isolation mount, whilst maintaining the same static load bearing capacity. Carella [4] and Ahn [5] have studied the static and dynamic behavior of HSLDS vibration isolator from the combination of positive and negative stiffness mechanism. Shaw [6] investigated steady state response of HSLDS isolator that is built by bistable composite plate.

Many studies have shown that dynamic motion of the HSLDS vibration isolator can be approximately described by the Duffing equation with linear and cubic stiffness terms. In fact, the resonance peak can be become large and jump phenomenon can occur at the resonant frequency as the base amplitude or force excitation increases [7]. It is evident that, the inherent nonlinearity of HSLDS should be controlled since it could result undesirable conditions in practical engineering. Lu et al. [8] and Donmez [9] have shown that nonlinear damping could be used to improve HSDLS isolator performance by reducing the transmissibility around the resonance frequencies without affecting its performance at high frequencies.

An alternative method for reducing structural vibrations is by active control. The concept of active control for modification of the system response has been reported in [10-11]. It has been stated that, the mass, damping and stiffness of the system can be independently modified depending on the type of feedback controller. In the case of damping modification, velocity feedback should be applied as the control strategy. [12-13].

Despite the growing interest in nonlinear isolator [1-3] and active vibration isolation [9- 10], only a few researchers have yet seriously examined the combination of HSLDS isolator with active control. In [14-15], the application of time-delayed linear displacement and cubic displacement feedback control of HSLDS isolator are studied. The authors showed that vibration isolation performance around the resonance frequency band under both force and base excitation is improved. However, there is still lack of literature in discussing the effect of active damping control on the performance of HSLDS isolator. In this paper, an investigation of the active damping effect on the HSLDS isolation system is presented. The harmonic balance method (HBM) is applied to derive an approximate solution of the amplitude frequency characteristic equation, and Floquet theory is employed to derive the boundary stability region on the obtained motion transmissibility. The effect of active damping on the motion transmissibility performance of the HSLDS isolation system is investigated analytically. All the obtained results are compared with an equivalent fully passive HSLDS isolation system.

The organization of this paper is as follows. First, the model of an actively damped HSLDS isolation system is presented, and its approximate solution to the equation of motion is derived in Section 1. In Section 2, the effect of active damping on the system transmissibility performance is investigated. In addition, the effect of active damping on the stability region of the plotted system transmissibility performance is also presented. Finally, conclusions are drawn in the final section.

MATHEMATICAL MODELLING OF AN ACTIVELY DAMPED HIGH-STATIC-LOW-DYNAMIC STIFFNESS ISOLATION SYSTEM

A single-degree-of-freedom (SDOF) HSLDS vibration isolator with active damping is shown in Figure 1. The system is subjected to base excitation, where $x(t)$ and $y(t)$ represent the mass and base displacement respectively. The model consists of a mass m which is mounted on a nonlinear spring (linear stiffness k_1 and cubic stiffness k_3), and viscous damper c in parallel with a secondary force f_s for active damping control purpose.

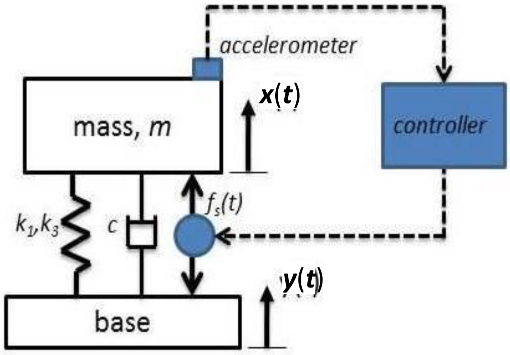

Figure 1. Single-degree-of-freedom (SDOF) model of an actively damped HLDS isolation system.

The equation of motion of the system which is subjected to harmonic base excitation can be expressed as

$$m\ddot{x} + c(\dot{x} - \dot{y}) + k_1(x - y) + k_3(x - y)^3 + f_s(t) = 0 \tag{1}$$

Note that, the control force f_s which is generated by the actuator is set to be proportional to the absolute velocity of the mass for the active damping control purpose.

$$f_s = c_{sky}\dot{x} \tag{2}$$

where c_{sky} is the control gains from the controller. Therefore, the equation of motion of the system can be written as

$$m\ddot{x} + c(\dot{x} - \dot{y}) + c_{sky}\dot{x} + k_1(x - y) + k_3(x - y)^3 = 0 \tag{3}$$

The motion of the base and the displacement of the mass are given as$y = Y\,cos(\,\omega t)$ and $x = X\,cos(\omega t + \theta)$ respectively. Y is the amplitude of the base excitation, X is the displacement amplitude of the mass (both are real positive values), ω is the excitation frequency, θ is the phase, and t is time.

By letting the relative displacement between the mass and the base be $u = x - y$, Eq. (3) can be simplified to become

$$m\ddot{u} + \left(c + c_{sky}\right)\dot{u} + k_1u + k_3u^3 = -m\ddot{y} - c_{sky}\dot{y} \tag{4}$$

which can be written in non-dimensional form as

$$\hat{u}'' + 2(\zeta + \zeta_{sky})\hat{u}' + \hat{u}(1 + K)\hat{u} + \alpha\hat{u}^3 = \Omega^2\,cos(\,\Omega\tau) + 2\zeta_{sky}\Omega\,sin(\,\Omega\tau) \tag{5}$$

where $\zeta = \frac{c}{2m\omega_n}$, $\zeta_{sky} = \frac{c_{sky}}{2m\omega_n}$, ${\omega_n}^2 = \frac{k_1}{m}$, $\alpha = \frac{k_3 Y^2}{k_1}$, $\Omega = \frac{\omega}{\omega_n}$, $\tau = \omega_n t$, $\hat{u}'' = \frac{\ddot{u}}{{\omega_n}^2 Y}$, $\hat{u}' = \frac{\dot{u}}{\omega_n Y}$, $\hat{u} = \frac{u}{Y}$, and a prime denotes differentiation with respect to non-dimensional time τ. Note that $\alpha$is a factor that determines the degree of nonlinearity of the system and is dependent on the magnitude of the base excitation Y, the linear stiffness k_1 and the cubic stiffness k_3. It is clear that the system defaults to a linear system if $\alpha = 0$. Meanwhile, ω_n is the natural frequency of the system when the amplitude of oscillations are small enough, so that the nonlinear term is negligible.

In this study the HBM to a first order expansion is applied with assumption that the response is harmonic and at the excitation frequency, such that

$$\hat{u} = \hat{U}\cos(\Omega\tau + \varphi) \tag{6}$$

where $\hat{U}$ and φ are the amplitude and phase of the relative displacement with respect to the base motion.By substituting Eq. (6) into Eq. (5), and neglecting the $cos\,3\,(\Omega\tau)$ term leads to a cubic equation for $\hat{U}^2$ with frequency dependent coefficients,

$$\frac{9}{16}\alpha^2\hat{U}^6 + \frac{3}{2}\alpha(1-\Omega^2)\hat{U}^4 + \left[(1-\Omega^2)^2 + 4(\zeta+\zeta_{sky})^2\Omega^2\right]\hat{U}^2 = \Omega^4 + 4\zeta_{sky}^2\Omega^2 \tag{7}$$

Eqn. (7) can be alternatively expressed as a quadratic in Ω^2 with amplitude dependent coefficients, where Ω is the frequency ratio,

$$\left(\hat{U}^2 - 1\right)\Omega^4 + \left[\left(4(\zeta+\zeta_{sky})^2 - 2\right)\hat{U}^2 - \frac{3}{2}\alpha\hat{U}^4 - 4\zeta_{sky}^2\right]\Omega^2 + \left(\frac{3}{4}\alpha\hat{U}^3 + \hat{U}\right)^2 = 0 \tag{8}$$

The two solutions to Eq. (8) are given by

$$\Omega = \sqrt{\frac{p_s \pm \sqrt{q_s}}{4(\hat{U}^2-1)}} \tag{9}$$

where

$$p_s = 3\alpha\hat{U}^4 + 4\left(1 - 2\left(\zeta+\zeta_{sky}\right)^2\right)\hat{U}^2 + 8\zeta_{sky}^2 \tag{10}$$

and

$$\begin{aligned} q_s = &\left(9\alpha^2 - 48\alpha\left(\zeta+\zeta_{sky}\right)^2\right)\hat{U}^6 \\ &+ \left(24\alpha\left(1-2{\zeta_{sky}}^2\right) - 64\left(\zeta+\zeta_{sky}\right)^2\left(1-\left(\zeta+\zeta_{sky}\right)^2\right)\right)\hat{U}^4 \\ &+ \left(16\left(1+4{\zeta_{sky}}^2\right) - 128{\zeta_{sky}}^2\left(\zeta+\zeta_{sky}\right)^2\right)\hat{U}^2 + 64{\zeta_{sky}}^4 \end{aligned} \tag{11}$$

of which only the real solutions, where they exist, are of interest.

RESULTS AND DISCUSSION

The Effect of Active Damping on the System Motion Transmissibility

In this study, the absolute motion transmissibility is used as the isolation performance measure. It is defined as the ratio between the mass and base absolute displacement, in steady-state vibration and at a given frequency excitation. Therefore, based on the absolute displacement, $\hat{x} = \hat{u} + \hat{y}$ which can be expressed as

$$\hat{x} = \hat{U}\, cos(\,\Omega\tau + \varphi) + cos(\,\Omega\tau) \tag{12}$$

The absolute motion transmissibility can be obtained by finding the magnitude of $\hat{x}$, such that

$$\hat{X} = \sqrt{1 + \hat{U}^2 + \frac{2\hat{U}^2\left(1 - \Omega^2 + \frac{3}{4}\alpha\hat{U}^2 - 4\zeta_{sky}(\zeta + \zeta_{sky})\right)}{\Omega^2 + 4\zeta_{sky}{}^2}} \tag{13}$$

For comparison purpose, the effect of passive damping and active damping in the absolutemotion transmissibility is demonstrated in Figure 2. with the nonlinearity value is set to $\alpha = 1.33\times10^{-4}$ to represent a system with mild nonlinearity [4]. The black solid line andthe bold blue line represents the motion transmissibility with passive damping ratio of $\zeta = 0.01$ (low damping value) and $\zeta = 0.1$ (high damping value) respectively. The results show that the increased passive damping ratio has reduced the resonance peak but hashad a detrimental effect on isolation at high frequencies.

In contrast, by applying active damping to the system, the resonance peak is suppressed without affecting the motion transmissibility performance at high frequencies. This is shown in the red dashed line in both plots which represents the active damping system. The total effective damping at the resonance in the active damping system is $\zeta = 0.1$, i.e. the sum of the active and passive damping values. This shows that by applying active damping to the system, an effective vibration isolations system which has a low resonance peak, low high frequency transmissibility and a large isolation range is achieved.

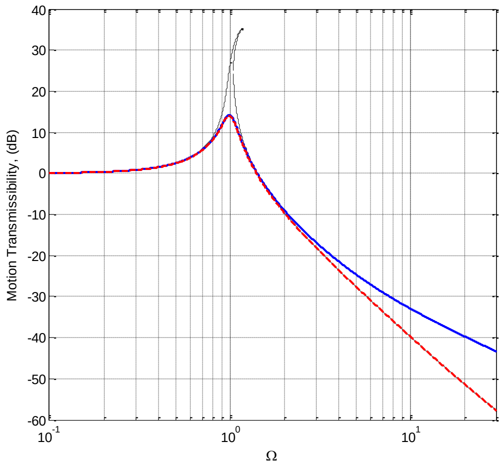

Figure 2. Motion transmissibility of passively and actively damped HSLDS isolationsystem $(\alpha = 1.33\times10^{-4})$. Passively damped; back solid line $(\zeta = 0.01)$, blue bold line $(\zeta = 0.1)$, and actively damped; red dashed line $(\zeta = 0.01$ and $\zeta_{sky} = 0.09)$.

The Effect of Active Damping on the System Stability

The stability of the periodic solution of the HSLDS isolation system in this work is based on Floquet theory. Therefore, the periodic solution of the system is assumed as

$$\hat{u} = \hat{U}\,cos(\Omega\tau + \varphi) + \varepsilon(\tau) \tag{14}$$

where ε is a small disturbance or perturbation applied to the response of the system. By substituting Eq. (14) into Eq. (5), and following the HBM will leads to a linear variational equation

$$\varepsilon'' + 2\left(\zeta + \zeta_{sky}\right)\varepsilon' + \left(1 + 3\alpha\hat{U}^2\cos^2\theta\right)\varepsilon = 0 \tag{15}$$

Assume the general solution of Eq. (15) to be a periodic function as

$$\varepsilon = e^{\lambda\tau}\left(P_1\cos\left(\Omega\tau + \varphi\right) + P_2\sin\left(\Omega\tau + \varphi\right)\right) \tag{16}$$

where P_1 and P_2 are Fourier coefficients of the first order expansion. The solution is unstable by definition if λ is positive, and stable if λ is negative. As a result, the solution to the linear variational equation tends to zero with increasing τ, if the value of λ has a negative real part, which corresponds to a stable periodic solution.

The effect of active damping in the stability of the system can be observed by substituting Eq. (16) into Eq. (15) such that the resulting expression may be written in matrix form as

$$\begin{pmatrix} \lambda^2 - \Omega^2 + 2\left(\zeta + \zeta_{sky}\right)\lambda + 1 + \frac{9}{4}\alpha\hat{U}^2 & 2\Omega\left(\lambda + \left(\zeta + \zeta_{sky}\right)\right) \\ -2\Omega\left(\lambda + \left(\zeta + \zeta_{sky}\right)\right) & \lambda^2 - \Omega^2 + 2\left(\zeta + \zeta_{sky}\right)\lambda + 1 + \frac{3}{4}\alpha\hat{U}^2 \end{pmatrix}\begin{pmatrix} P_1 \\ P_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{17}$$

where the determinant of Eq. (17) can be expressed as

$$\lambda^4+4\left(\zeta+\zeta_{sky}\right)\lambda^3+\left(4\left(\zeta+\zeta_{sky}\right)^2+2+2\Omega^2+3\alpha\hat{U}^2\right)\lambda^2+$$
$$\left(2\left(\zeta+\zeta_{sky}\right)\left(2+2\Omega^2+3\alpha\hat{U}^2\right)\right)\lambda+\left(1-\Omega^2+\frac{9}{4}\alpha\hat{U}^2\right)\left(1-\Omega^2+\frac{3}{4}\alpha\hat{U}^2\right)+4\left(\zeta+\zeta_{sky}\right)^2\Omega^2=0 \tag{18}$$

Hence, the solution for λ is given by

$$\lambda=-\left(\zeta+\zeta_{sky}\right)\pm\sqrt{\left(\left(\zeta+\zeta_{sky}\right)^2-\Omega^2-1-\frac{3}{2}\alpha\hat{U}^2\right)\pm\sqrt{4\Omega^2\left(1-\left(\zeta+\zeta_{sky}\right)^2+\frac{3}{2}\alpha\hat{U}^2\right)+\left(\frac{3}{4}\alpha\hat{U}^2\right)^2}} \tag{19}$$

Subsequently, the boundary of the stability region can be obtained by letting $\lambda=0$ in Eq. (19) and solved for Ω to yield

$$\Omega_{unst}=\sqrt{\left(1-2\left(\zeta+\zeta_{sky}\right)^2+\frac{3}{2}\alpha\hat{U}^2\right)\pm2\sqrt{\left(\zeta+\zeta_{sky}\right)^2\left(\left(\zeta+\zeta_{sky}\right)^2-1-\frac{3}{2}\alpha\hat{U}^2\right)+\left(\frac{3}{8}\alpha\hat{U}^2\right)^2}} \tag{20}$$

Generally, the plots of Eq. (20) can be defined as stability parabola curves. By plotting it over the transmissibility curve of given by Eq. (13), the boundary of stability region can be observed as demonstrated in Figure 3.

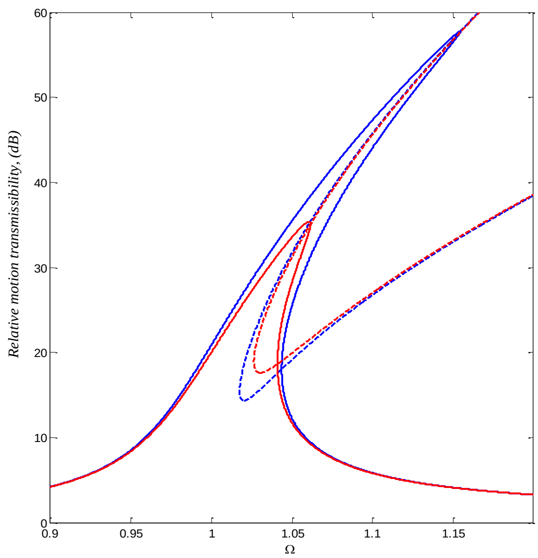

Figure 3. Stability parabola curves in the relative motion transmissibility plot of an actively damped HSLDS isolation system $(\alpha=1.33\times10^{-4},\zeta=0.01)$.Relative motion transmissibility curve; passive system (blue solid line), actively damped system with $\zeta_{sky}=0.005$ (red solid line), stability parabola curves; passive system (blue dashed line), actively damped system with $\zeta_{sky}=0.005$ (red dashed line).

By putting the values of $U^{\hat{}}$ and Ω into Eq. (19), the value of the real part of λ is only positive, when it is in the enclosed curve of the dashed line. This represents the instability region of the system. For example, for a fully passive system, the instability region is enclosed by the blue dashed line. Meanwhile, for an actively damped system with $\zeta_{sky} = 0.005$, the instability region is enclosed by the red dashed line. In Figure 4 and Figure 5, the instability region for the respective fully passive and active damping system,are presented in the absolute motion transmissibility plot with the shaded area as determined in Eq. (20). Note that, the instability region decreases when the active damping is applied, which demonstrates a more dynamically stable system.

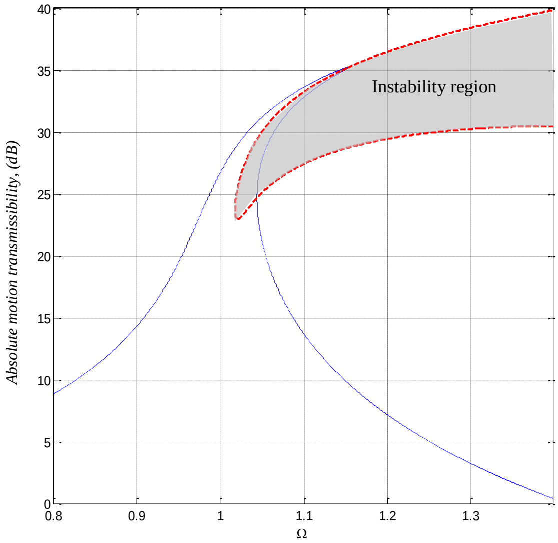

Figure 4. Instability region in the absolute motion transmissibility plot of a HSLDS isolation system $(\alpha = 1.33\times10^{-4}, \zeta = 0.01)$. The red dashed line represents unstable parabola, where the instability region is shaded.

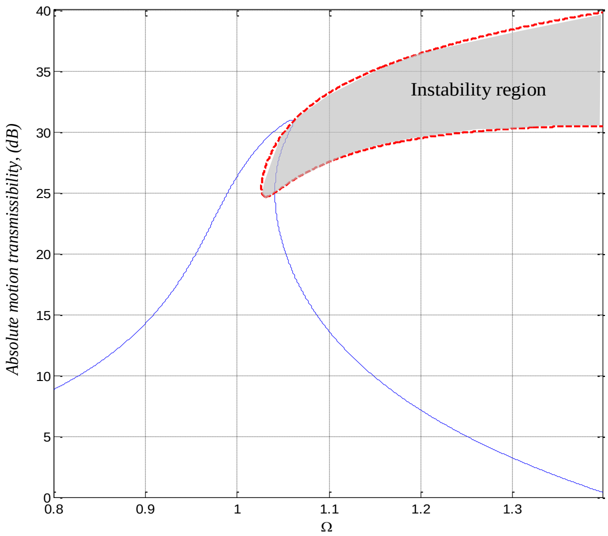

Figure 5. Instability region in the absolute motion transmissibility plot of an actively damped HSLDS isolation system $(\alpha = 1.33\times10^{-4}, \zeta = 0.01, \zeta_{sky} = 0.005)$. The red dashed line represents unstable parabola, where the instability region is shaded.

By referring to both figures (Figure 4 and Figure 5), it can be seen that there are two intersection points that occur between the transmissibility curves with the corresponding parabola stability curves. The first intersection point occurs at the peak of the transmissibility curve, which corresponds to the jump down frequency. Meanwhile, the second intersection point happens on the non-resonant branch which represents the jump up frequency. Therefore, the region between the jump up and jump down frequencies which is enclosed by the parabola plot is the unstable equilibrium state. When active damping is applied, the transmissibility peak is reduced and will results in a reduced jump down frequency. This is illustrated in Figure 5, where an active damping value of 0.005 is applied to the system. In addition, by comparing Figure 4 and Figure 5, it can be observed that the active damping changes the jump down frequency drastically, but with only very slight changes for the jump up frequency.

It is also interesting to note that, if the total damping of the system is negative where $\zeta + \zeta_{sky} < 0$, the real part of the solution of Eq. (19) will become positive. As a result, the system becomes totally unstable. The boundary of stability that determined by the parabola stability curves in Eq. (20), then has the instability region inside and outside the enclosed curves.

CONCLUSIONS

In this paper, the effect of active damping on the response of HSLDS isolator subjected to harmonic excitation is studied based on SDOF HSLDS isolation model. The active damping was generated by a control force from an actuator which is set to be proportional to the absolute velocity of the isolated mass. The results show that by applying active damping to the system, large bending resonance curve of HSLDS isolator that detriments isolation performance could be suppressed, without compromising the isolation bandwidth at high frequencies. Therefore, an actively damped HSLDS isolator could be an effective vibration isolations system which has a low resonance peak, low high frequency transmissibility and a large isolation range. This contrasted with the employment of passive damping where the increment of passive damping ratio has reduced the resonance peak but has had a detrimental effect on isolation at high frequencies. In addition, the determined stability parabola curves on the motion transmissibility plot has shown that the instability region decreases when active damping is applied, which demonstrates a more dynamically stable system.

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